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January 15, 2019 8:19 PM

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Pre Calculus 11 Ch3/4 Lesson 8b: Solving Max and Minimum Problems:

1. Two numbers have a difference of 10. Their product is a minimum. Determine the numbers

① Let x, $x+10$ be the two numbers

② Product (min) $= (x)(x+10)$. 3 ways to find the vertex

$\boxed{\text{C.T.S.}}$ $P = (x)(x+10)$
$= x^2+10x$
$= x^2+10x+25-25$
$= (x^2+10x+25)-25$
$= (x+5)^2-25$

The vertex is $(-5,-25)$ so $x=-5$, the product is -25, the other number is 5.

$\boxed{\text{X.A.V.}}$ $P=(x)(x+10)$
$0=(x)(x+10)$
$x=0 \quad x=-10$
Avg: $x = \frac{0+(-10)}{2} = -5$.
$P=(-5)(-5+10)$
$=-25$ ← Minimum Product.

Min product occurs when $x=-5$.

$\boxed{x=-\frac{b}{2a}}$

$P=(x)(x+10)$ $a=1$
$=x^2+10x$ $b=10$
$x_{min} = \frac{-10}{2(1)} = -5$.
So $P=-5(-5+10)$
$=-25$.

2. The sum of two natural numbers is 12. Their product is a maximum. Determine the numbers

① Sum is 12 → $x+y=12$ So the two numbers are x : $12-x$
$y=12-x$

② Product is a maximum
$P=(x)\cdot(y)$
$=(x)(12-x)$.

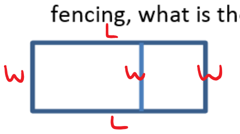
Go thru the 3 ways to find the vertex:
① C.T.S.
② X.A.V.
③ $-b/2a$.

i) CTS.
$P=(x)(12-x)$
$=-x^2+12x$
$=-(x^2-12x)$
$=-(x^2-12x+36)+36$
$P=-(x-6)^2+36$.
Vertex $(6,36)$
So when $x=6$, the max. product is 36. The other number 'y' is also 6.

ii) X.A.V.
$P=(x)(12-x)$
$0=(x)(12-x)$
$x=0 \quad x=12$.
$x_{max} = \frac{0+12}{2} = 6$.
$P=6(12-6)$
$P=36$. When $x=6$
The max. product will be 36.

iii) $x_{vertex} = -b/2a$.
$P=x(12-x)$ $a=-1$
$=-x^2+12x$ $b=12$
$x_{max} = \frac{-12}{2(-1)} = 6$.
$P=6(12-6)$
$=36$

3. A rectangular area is enclosed by a fence and separated into 2 rectangular regions as shown. With 800m of fencing, what is the maximum area that could be enclosed. Find the dimensions of the enclosed area.



① $P=2L+3W$
$800=2L+3W$
$800-3W=2L$
$\boxed{400-1.5W=L}$

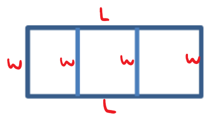
② Area $=W(L)$
$A=W(400-1.5W)$
Now find the vertex.
i) C.T.S.
ii) XAV.
iii) $-b/2a$.

C.T.S.
$A=-1.5W^2+400W$
$A=-1.5\left(W^2-\frac{800}{3}W+\frac{160000}{9}-\frac{160000}{9}\right)$
$A=-1.5\left(W^2-\frac{800}{3}W+\frac{160000}{9}\right)+\frac{80000}{3}$
$A=-1.5\left(W-\frac{400}{3}\right)^2+\frac{80000}{3}$.
Vertex $\left(\frac{400}{3}, \frac{80000}{3}\right)$

The max area is $\frac{80000}{3}$ and it occurs when the width is equal to $\frac{400}{3}$, the length is 200.

X.A.V.
$A=W(400-1.5W)$
$0=W(400-1.5W)$
$W=0 \quad W=\frac{800}{3}$
$x_{vertex} = \left(0+\frac{800}{3}\right)\times\frac{1}{2} = \boxed{\frac{400}{3}}$
$A=\frac{400}{3}\left(400-\frac{3}{2}\left(\frac{400}{3}\right)\right)$
$A=\frac{400}{3}(200) = \boxed{\frac{80000}{3}}$ (Max area)

4. Suppose the rectangular fence is to be separated into 3 rectangular regions as shown. Again, with 800m of fencing, find the maximum area that could be enclosed. Find the dimensions of the enclosed area.



① Perimeter $=2L+4W$
$800=2L+4W$
$800-4W=2L$
$\boxed{400-2W=L}$

② Area $=L\times W$
$A=(400-2W)W$

3 methods
① $-b/2a$
② XAV.
③ CTS.

i) $-b/2a$.
$A=(400-2W)(W)$
$=-2W^2+400W$.
$a=-2 \quad b=400$
$x_{max} = -\frac{b}{2a} = \frac{-400}{2(-2)}$
$=100$
$A=(400-2(100))(100)$
$=(400-200)(100)$
$=(200)(100)$
$=20000$

The width of 100m will generate a max area of 20000m^2. The length is 200.

ii) X.A.V.
$A=(400-2W)(W)$
$0=(400-2W)(W)$
$W=200 \quad W=0$
$W_{Avg} = \frac{200+0}{2} = 100$.
This width will generate the max area.
$A=(400-2(100))(100)$
$=(200)(100)$
$=20000\text{m}^2$
$W=100$, $A=20000$
$L=200$

iii) CTS.
$A=(400-2W)(W)$
$=-2W^2+400W = -2(W^2-200W)$
$=-2(W^2-200W+10000-10000)$
$=-2(W^2-200W+10000)+20000$
$=-2(W-100)^2+20000$
Vertex (W,A)
$=(100,20000)$
Width $=100$m, Max area $=20000$
Length $=\frac{A}{W}$
$=200$m.

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5. A company that charters a boat for tours around Vancouver Island can sell 200 tickets at $50 each. For every $10 increase in the ticket price, 5 fewer tickets will be sold. What selling price will provide the maximum revenue? What is the maximum revenue?

① $\frac{Q - Q_0}{P - P_0} = \frac{\Delta Q}{\Delta P}$

$\frac{Q-200}{P-50} = \frac{-5}{10}$

$Q - 200 = -\frac{1}{2}(P-50)$

$Q = -\frac{1}{2}P + 25 + 200$

$\boxed{Q = -\frac{1}{2}P + 225}$

② $Rev = P \times Q$

$R = P(-\frac{1}{2}P + 225)$

$R = -\frac{1}{2}P^2 + 225P$

$R = -\frac{1}{2}(P^2 - 450P)$

$R = -\frac{1}{2}(P^2 - 450P + 225^2 - 225^2)$

$R = -\frac{1}{2}(P^2 - 450P + 225^2) + \frac{1}{2}(225^2)$

$R = -\frac{1}{2}(P - 225)^2 + 25,312.5$

2nd Method

$R = P \times Q$

$R = (50 + 10x)(200 - 5x)$

$R = 10(5+x)5(40-x)$

$R = 50(5+x)(40-x)$

$0 = 50(5+x)(40-x)$

$x = -5$ $x = 40$

$x = \frac{-5+40}{2} = 17.5$

So $R = (50 + 10(17.5))(200 - 5(17.5))$

$= (225)(112.5)$

$= \$25,312.50$

The selling price is $225.00 and the max rev. is $25,312.50

6. A Broadway musical sells 400 tickets each day at $30 per ticket. For every increase of $3.00, they lose 20 sales. What should their ticket price be to yield the maximum revenue?

① $\frac{Q-400}{P-30} = \frac{-20}{3}$

$Q - 400 = -\frac{20}{3}(P-30)$

$Q = -\frac{20}{3}P + 200 + 400$

$Q = -\frac{20}{3}P + 600$

② $R = P \times Q$

$= P(-\frac{20}{3}P + 600)$

$= -\frac{20}{3}P^2 + 600P$

$a = -\frac{20}{3}$ $b = 600$ $c = 0$

$P = \frac{-600}{2(-\frac{20}{3})} = \frac{600(3)}{40} = 45$

$R = 45(-\frac{20}{3} \cdot 45 + 600)$

$= 45(-300 + 600)$

$= \$13,500$

$R = P \times Q$

$R = (400 - 20x)(30 + 3x)$

$R = 20(20-x)3(10+x)$

$R = 60(20-x)(10+x)$

$0 = 60(20-x)(10+x)$

$x = 20$ $x = -10$

$x_{Avg} = \frac{20 + (-10)}{2} = \frac{10}{2} = 5$

$R = (400 - 20(5))(30 + 3(5))$

$= (400 - 100)(30 + 15)$

$= (300)(45)$

$= \$13,500$

The price that generates the max rev. is

$P = 45$ $R_{(max)} = \$13,500$

$Q = 300$

7. A company sells its bikes at $300 each and can sell 70 in a season. For every $25 increase in the price, the number sold drops by 10. What price will yield the maximum revenue?

① $\frac{Q-70}{P-300} = \frac{-10}{25}$

$Q - 70 = -\frac{10}{25}(P-300)$

$Q - 70 = -\frac{2}{5}P + 120$

$Q = -\frac{2}{5}P + 190$

② $R = P \times Q$

$= P(-\frac{2}{5}P + 190)$

$= -\frac{2}{5}P^2 + 190P$

③ $a = -\frac{2}{5}$ $b = 190$

$P_{max} = \frac{-190}{2(-\frac{2}{5})}$

$= 237.50$

$R = P(-\frac{2}{5}P + 190)$

$R = (237.50)(-\frac{2}{5} \cdot 237.50 + 190)$

$R = (237.50)(95)$

$R = 22,562.50$

$P = 237.50$ $Q = 95$

$R_{MAX} = 22,562.50$

① $R = P \times Q$

$R = (300 + 25x)(70 - 10x)$

$R = 25(12 + x)(10)(7 - x)$

$R = 250(12 + x)(7 - x)$

$0 = 250(12 + x)(7 - x)$

$x = -12$ $x = 7$

$x_{Avg} = \frac{-12 + 7}{2} = -2.5$

$R = (300 + 25(-2.5))(70 - 10(-2.5))$

$= (300 - 62.5)(70 + 25)$

$= (237.50)(95)$

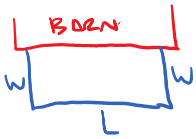
$= 22,562.50$

$P = \$237.50$

$Q = 95$

$R_{max} = 22,562.50$

8. A farmer wants to make a rectangular corral by using his barn wall as one of the sides of the corral. If the farmer has only 60m of fence, what length for the rectangular corral would maximize the area?



① $P = 2W + L$

$60 = 2W + L$

$\boxed{60 - 2W = L}$

② $Area = L \times W$

$A = (60 - 2W)(W)$

$A = -2W^2 + 60W$

$A = -2(W^2 - 30W)$

$A = -2(W^2 - 30W + 225 - 225)$

$A = -2(W - 15)^2 + 450$

$W = 15$, max area $= 450$

$L = 450 \div 15 = 30m$

X.A.V.

$A = (60 - 2W)(W)$

$0 = (60 - 2W)(W)$

$W = 30$ $W = 0$

$W_{Avg} = \frac{30 + 0}{2} = 15m$

$A = (60 - 2(15))(15)$

$= (60 - 30)(15)$

$A = 450m$

$-b/2a$

$A = (60 - 2W)(W)$

$= -2W^2 + 60W$

$a = -2$ $b = 60$

$W = \frac{-60}{2(-2)} = 15m$

$A = (60 - 30)(15)$

$A = 450m$

9. Challenge: This one is super hard. The parabola $y = f(x) = x^2 + bx + c$ has vertex "P" and the parabola $y = g(x) = -x^2 + dx + e$ has vertex "Q", where "P" and "Q" are distinct points. The two parabolas also intersect at "P" and "Q". Prove that $2(e - c) = bd$.

① PARABOLA #1 $y = f(x)$

$f(x) = x^2 + bx + c$

FIND THE VERTEX IN TERMS OF B & C.

$y = (x^2 + bx) + c$

$y = (x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4}) + c$

$y = (x^2 + bx + \frac{b^2}{4}) - \frac{b^2}{4} + c$

$y = (x - \frac{b}{2})^2 - \frac{b^2}{4} + c.$

vertex P $(-\frac{b}{2}, c - \frac{b^2}{4})$

② PARABOLA $y = g(x)$

$g(x) = -x^2 + dx + e.$

FIND THE COORDINATES OF THE PARABOLA IN TERMS OF 'd' AND 'e'.

$y = -x^2 + dx + e$

$y = -(x^2 - dx) + e$

$y = -(x^2 - dx + \frac{d^2}{4} - \frac{d^2}{4}) + e$

$y = -(x^2 - dx + \frac{d^2}{4}) + \frac{d^2}{4} + e$

$y = -(x - \frac{d}{2})^2 + \frac{d^2}{4} + e$

vertex Q: $(\frac{d}{2}, e + \frac{d^2}{4})$

③ WE ARE TOLD THAT BOTH PARABOLAS INTERSECT AT EACH OTHER'S VERTEX. THAT MEANS WE CAN PLUG THE COORDINATES OF THE VERTEX INTO THE OTHER EQUATION:

ie: PLUG 'P' INTO $y = g(x)$ OR PLUG 'Q' INTO $y = f(x)$.

$g(x) = -x^2 + dx + e$

$c - \frac{b^2}{4} = -(-\frac{b}{2})^2 + d(\frac{-b}{2}) + e$

$c - \cancel{\frac{b^2}{4}} = -\cancel{\frac{b^2}{4}} - \frac{db}{2} + e$

$c = e - \frac{db}{2}$

$\frac{db}{2} = e - c$

$db = 2(e - c)$

$bd = 2(e - c)$

✓

$f(x) = x^2 + bx + c$

$e + \frac{d^2}{4} = (\frac{d}{2})^2 + b(\frac{d}{2}) + c$

$e + \cancel{\frac{d^2}{4}} = \cancel{\frac{d^2}{4}} + \frac{bd}{2} + c$

$e = \frac{bd}{2} + c$

$e - c = \frac{bd}{2}$

$2(e - c) = bd$

✓

